Mediation and Causal Mechanisms: A Continuous-Time Approach

Oisín Ryan & Ellen Hamaker

Department of Methodology and Statistics, Utrecht University

Overview

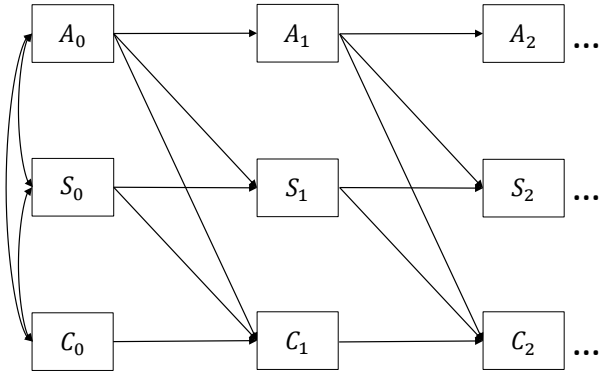
- Context
  - Data: ESM studies
  - Model: VAR(1)
  - Substantive interest: Mediation

Overview

- Context
  - Data: ESM studies
  - Model: VAR(1)
  - Substantive interest: Mediation
- Summary
  - **Traditional VAR(1)** models - misleading interpretation of causal structure
  - **Continuous-Time** models - alternative conceptual framework

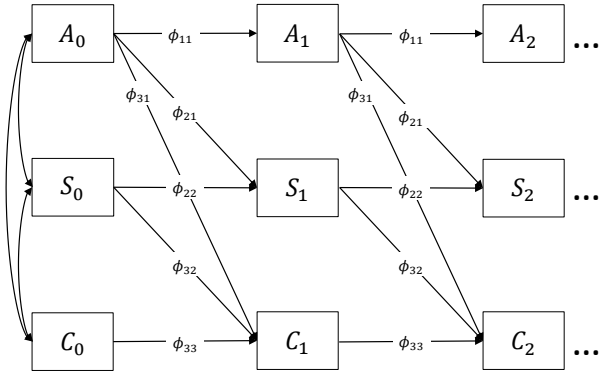
VAR(1) Model

$$\boldsymbol{Y}_\tau = \boldsymbol{\Phi}\boldsymbol{Y}_{\tau-1} + \boldsymbol{\epsilon}_\tau$$



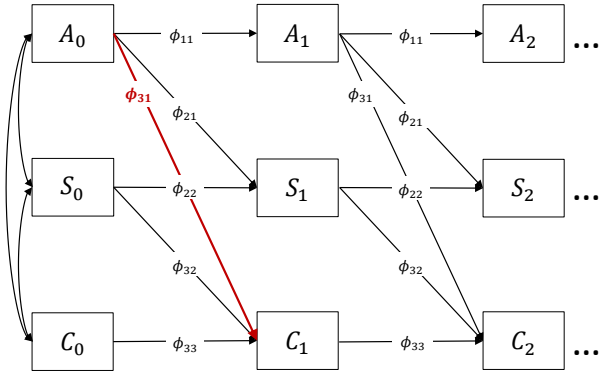
VAR(1) Model

$$\boldsymbol{Y}_\tau = \boldsymbol{\Phi}\boldsymbol{Y}_{\tau-1} + \boldsymbol{\epsilon}_\tau$$



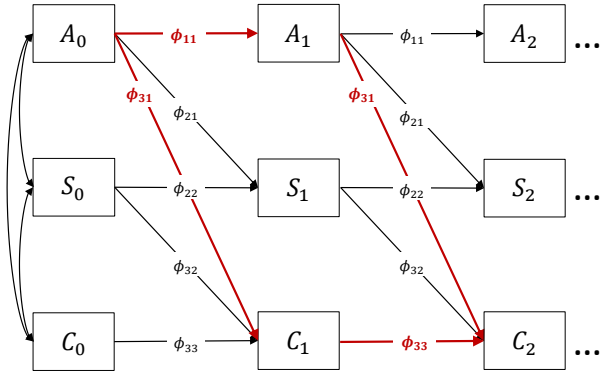
VAR(1) Model: Direct effects

$$\boldsymbol{Y}_\tau = \boldsymbol{\Phi}\boldsymbol{Y}_{\tau-1} + \boldsymbol{\epsilon}_\tau$$



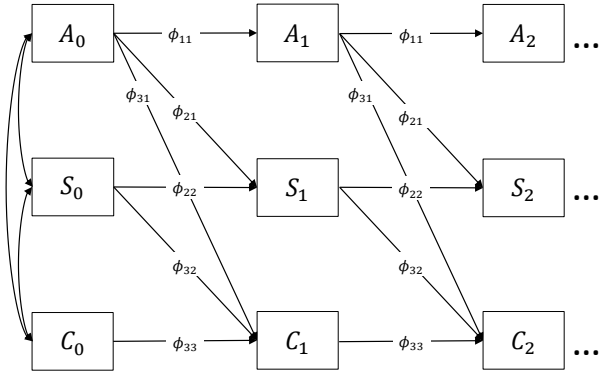
VAR(1) Model: Direct effects

$$\boldsymbol{Y}_\tau = \boldsymbol{\Phi}\boldsymbol{Y}_{\tau-1} + \boldsymbol{\epsilon}_\tau$$



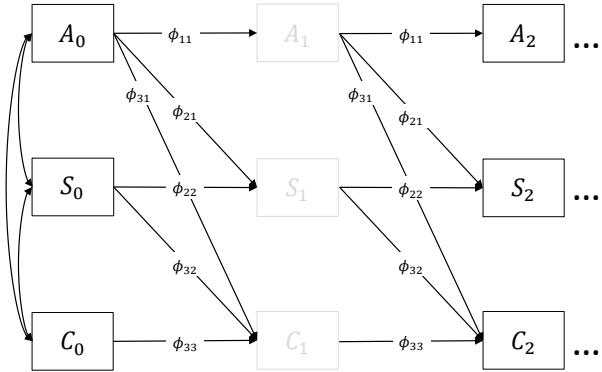
The Time-Interval Problem (cf. Gollob & Reichardt, 1987)

$$\boldsymbol{Y}_\tau = \boldsymbol{\Phi}(\Delta t = 1)\boldsymbol{Y}_{\tau-1} + \boldsymbol{\epsilon}_\tau$$



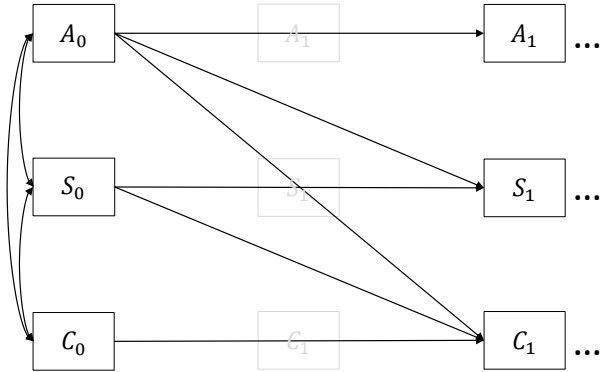
The Time-Interval Problem (cf. Gollob & Reichardt, 1987)

$$\boldsymbol{Y}_\tau = \boldsymbol{\Phi}(\Delta t = 1)\boldsymbol{Y}_{\tau-1} + \boldsymbol{\epsilon}_\tau$$



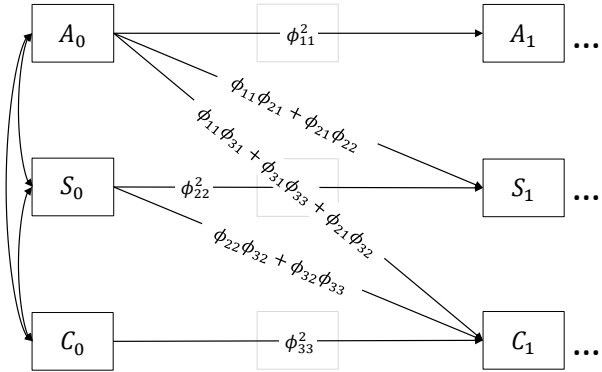
The Time-Interval Problem (cf. Gollob & Reichardt, 1987)

$$\boldsymbol{Y}_\tau = \boldsymbol{\Phi}(\Delta t = 2)\boldsymbol{Y}_{\tau-1} + \boldsymbol{\epsilon}_\tau$$



The Time-Interval Problem (cf. Gollob & Reichardt, 1987)

$$\mathbf{\Phi}(\Delta t = 2) = \mathbf{\Phi}(\Delta t = 1)^2$$



Implications of the Time-Interval Problem

1. For a uniform time-interval Δt
   - $\mathbf{\Phi} = \mathbf{\Phi}(\Delta t)$ *only for the given time-interval*

Implications of the Time-Interval Problem

1. For a uniform time-interval Δt
   - $\mathbf{\Phi} = \mathbf{\Phi}(\Delta t)$ *only for the given time-interval*
2. Unevenly spaced observations
   - $\mathbf{\Phi} \neq \mathbf{\Phi}(\Delta t)$ *for any time-interval*

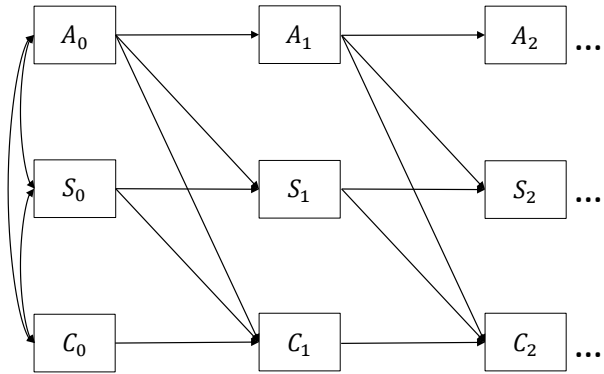
Implications of the Time-Interval Problem

1. For a uniform time-interval Δt
   - $\mathbf{\Phi} = \mathbf{\Phi}(\Delta t)$ *only for the given time-interval*
2. Unevenly spaced observations
   - $\mathbf{\Phi} \neq \mathbf{\Phi}(\Delta t)$ *for any time-interval*
   - DSEM can correct for this - VARIABLE: TINTERVAL= t(Δt)
   - See Asparouhov, Hamaker & Muthen (2017) Technical Report

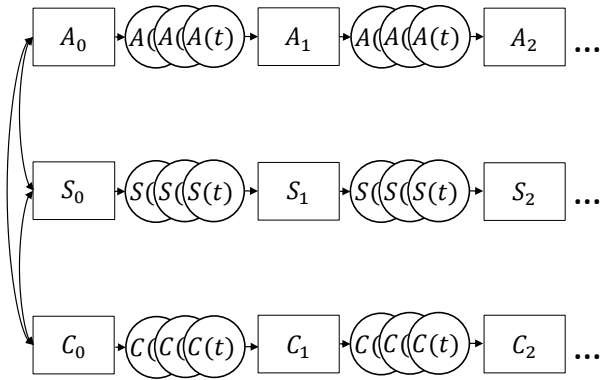
Implications of the Time-Interval Problem

1. For a uniform time-interval Δt
   - $\mathbf{\Phi} = \mathbf{\Phi}(\Delta t)$ *only for the given time-interval*
2. Unevenly spaced observations
   - $\mathbf{\Phi} \neq \mathbf{\Phi}(\Delta t)$ *for any time-interval*
   - DSEM can correct for this - VARIABLE: TINTERVAL= t(Δt)
   - See Asparouhov, Hamaker & Muthen (2017) Technical Report
3. Interpretation of $\mathbf{\Phi}$ parameters as direct effects is questionable
   - **Continuous Time Modelling** offers an alternative framework for path specific effects - (Deboeck & Preacher, 2015; Aalen et al. 2012).

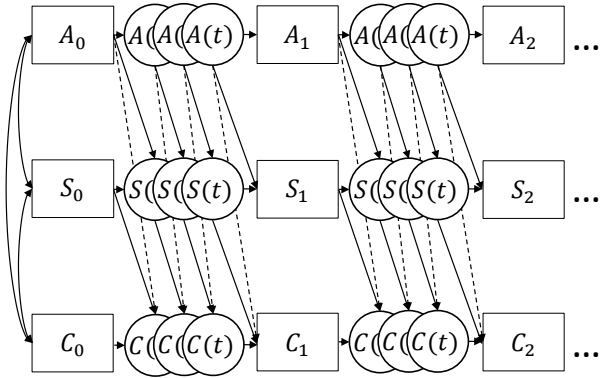
From discrete-time to continuous-time



From discrete-time to continuous-time

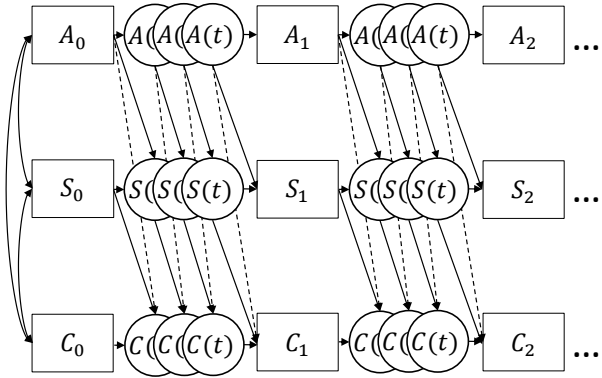


From discrete-time to continuous-time

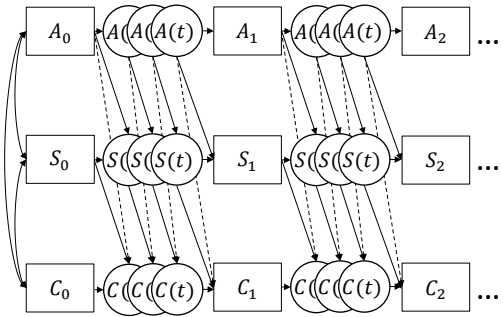


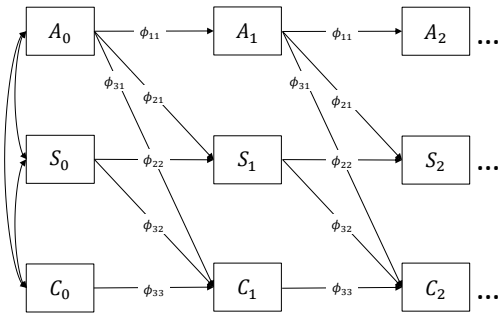
From discrete-time to continuous-time

$$\boldsymbol{Y}_\tau = \boldsymbol{e}^{\boldsymbol{A}\Delta t}\boldsymbol{Y}_{\tau-1} + \boldsymbol{\epsilon}_\tau$$



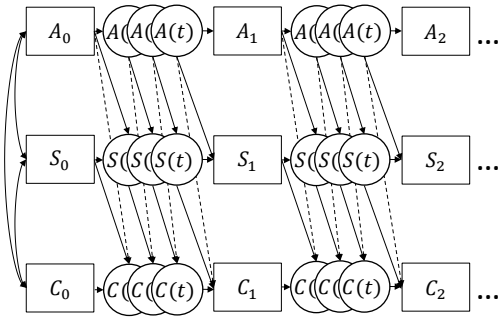
Direct effects: CT vs DT

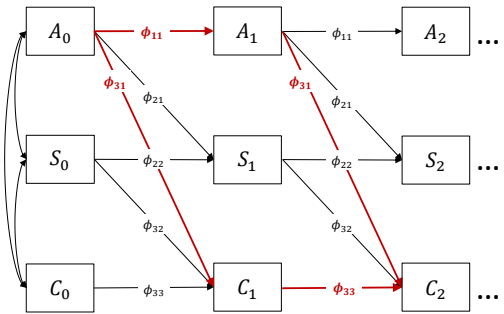




Direct effects: CT vs DT

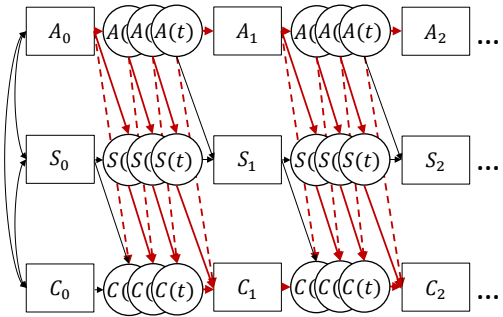
"Direct" Effect $= \phi_{11}\phi_{31} + \phi_{31}\phi_{33}$

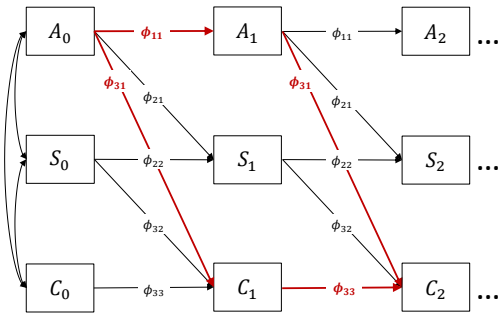




Direct effects: CT vs DT

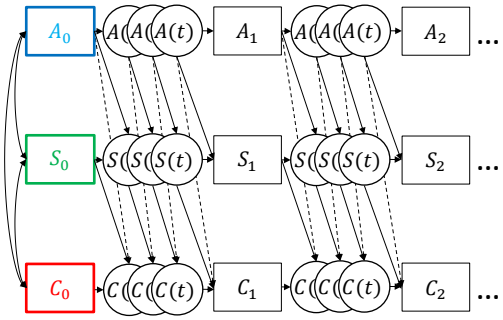
"Direct" Effect $= \phi_{11}\phi_{31} + \phi_{31}\phi_{33}$

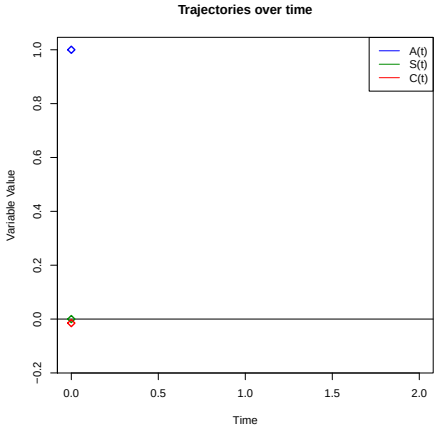




Direct effects as hypothetical experiments

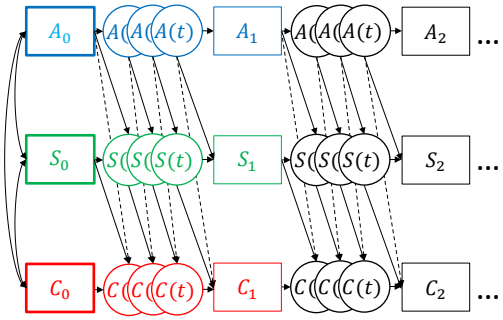
$$\text{“Direct” Effect} = \phi_{11}\phi_{31} + \phi_{31}\phi_{33}$$

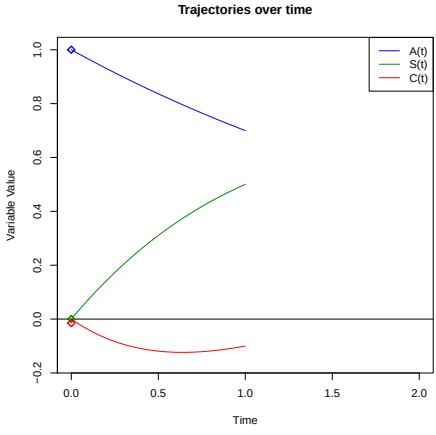




Direct effects as hypothetical experiments

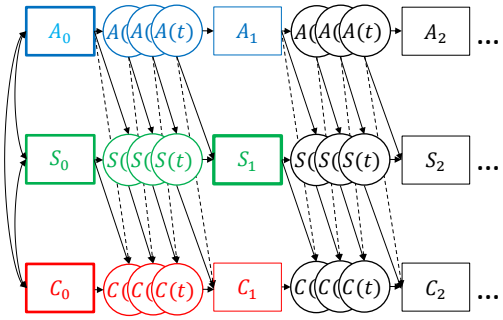
$$\text{“Direct” Effect} = \phi_{11}\phi_{31} + \phi_{31}\phi_{33}$$

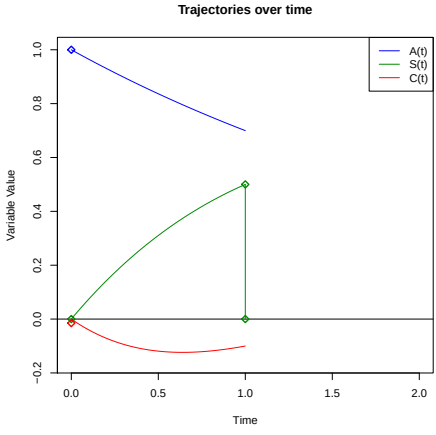




Direct effects as hypothetical experiments

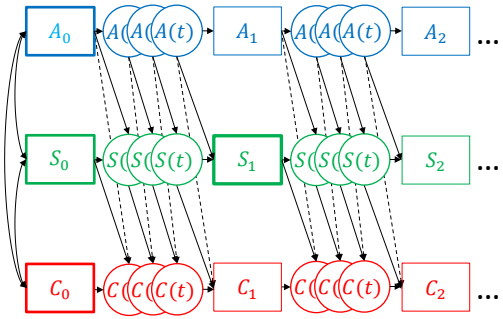
$$\text{“Direct” Effect} = \phi_{11}\phi_{31} + \phi_{31}\phi_{33}$$

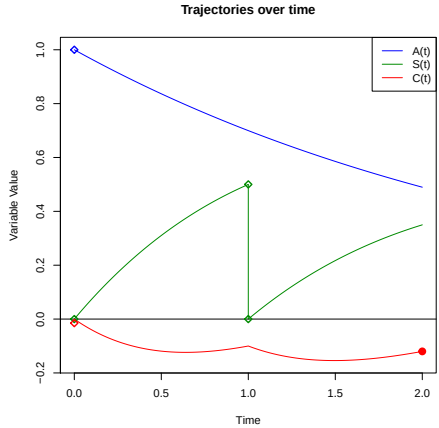




Direct effects as hypothetical experiments

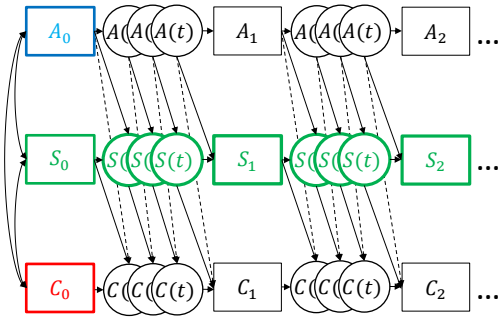
$$\text{“Direct” Effect} = \phi_{11}\phi_{31} + \phi_{31}\phi_{33}$$

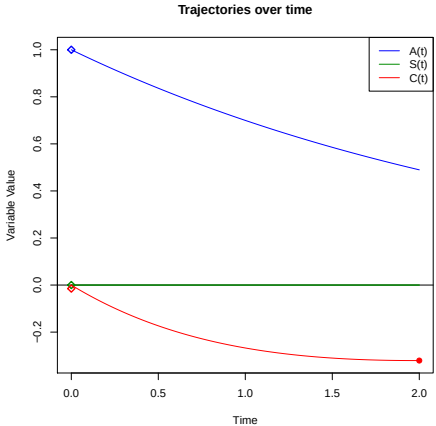




Direct effects as hypothetical experiments

CT model Direct Effect





Estimating the model

- Numerous promising R packages for CT estimation
  - ctsem - Driver, Voelkle, Oud
  - GLLA and LDE through OpenMx - Boker and colleagues

Estimating the model

- Numerous promising R packages for CT estimation
  - ctsem - Driver, Voelkle, Oud
  - GLLA and LDE through OpenMx - Boker and colleagues
- DSEM is promising as a method of indirectly estimating the CT model
  - First estimate $\mathbf{\Phi}(\Delta t)$
  - Then "solve" for the underlying CT matrix $\boldsymbol{A}$
- Simulation studies are needed to fully flesh out the possibilities (in progress!)

Conclusion

- DSEM allows us to fit a wide range of time series models
- To make further progress we need to examine how we conceptualise dynamic systems, and how we interpret the parameters of these models

Contact Details

- https://dml.sites.uu.nl/
- o.ryan@uu.nl